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ABSTRACT

Faced with a nonstandard, complicated practical problem in statistical inference, the applied statistician sometimes must use asymptotic approximations in order to compute standard errors and confidence intervals and to test hypotheses. This usually requires that he derive formulas for one or more asymptotic sampling variances (and covariances) for one or more statistics. He must then compute the numerical value of an estimate of some function of these variances and covariances. If a statistic is a nonlinear function of more than two or three sample statistics, the mathematical derivation of the necessary variance (and covariance) formulas may be burdensome, or even prohibitive. The purpose of the present paper is to call attention to computer program LASAHT that computes estimated asymptotic sampling variances and covariances numerically and carries out hypothesis tests without need for the statistician to derive formulas for them. (Author/RC)

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AUTOMATED HYPOTHESIS TESTS AND STANDARD ERRORS FOR NONSTANDARD PROBLEMS

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Introduction

Faced with a nonstandard, complicated practical problem in statistical inference, the applied statistician sometimes must use asymptotic approximations in order to compute standard errors and confidence intervals and to test hypotheses. This usually requires that he derive formulas for one or more asymptotic sampling variances (and covariances) for one or more statistics $\hat{\xi}_1, \hat{\xi}_2, \ldots$. He must then compute the numerical value of an estimate of some function of these variances and covariances.

If $\hat{\xi}_1$ is a nonlinear function of more than two or three sample statistics, the mathematical derivation of the necessary variance (and covariance) formulas may be burdensome, or even prohibitive. The purpose of the present paper is to call attention to computer program LASAHT that computes estimated asymptotic sampling variances and covariances numerically and carries out hypothesis tests without need for the statistician to derive formulas for them. AUTEST, written by Martha Stocking, and instructions for its use (Stocking and Lord, 1973) are available from the authors.

Asymptotic Variances and Covariances

Let $\hat{\xi} \equiv \xi(\underset{\sim}{t})$ be a differentiable function of sample statistics denoted by the vector $\underset{\sim}{t} \equiv \{t_u\} \equiv \{t_u(\underset{\sim}{X})\}$. Denote the expectation of $\underset{\sim}{t}$ by $\underset{\sim}{\tau} \equiv \{\tau_u\}$. If the t_u have variances of order N^{-1} , where N is the

sample size, then the asymptotic variance of $\hat{\xi}$ if finite is given (Kendall and Stuart, 1958, eq. 10.12) by

$$\text{Var } \hat{\xi} = \sum_u \sum_v \frac{\partial \xi(\tau)}{\partial \tau_u} \frac{\partial \xi(\tau)}{\partial \tau_v} \text{Cov}(t_u, t_v) \tag{1}$$

provided (1) is nonzero.

A consistent estimator $\hat{\sigma}^2_{\hat{\xi}}$ is usually obtained by substituting $\underset{\sim}{t}$ for $\underset{\sim}{\tau}$ in (1):

$$\hat{\sigma}^2_{\hat{\xi}} = \sum_u \sum_v \frac{\partial \xi(t)}{\partial t_u} \frac{\partial \xi(t)}{\partial t_v} c\phi v(t_u, t_v) \tag{2}$$

where

$$c\phi v(t_u, t_v) \equiv \text{Cov}(t_u, t_v)\Big|_{\underset{\sim}{\tau} = \underset{\sim}{t}} .$$

If (1) is a rational function, consistency follows from a proposition due to Slutsky (Cramér, 1946, p. 255). A similar result is deduced from different assumptions by Rao (1965, p. 322). The covariance between two functions $\hat{\xi}_a$ and $\hat{\xi}_b$ is similarly estimated from

$$\hat{\sigma}_{\hat{\xi}_a \hat{\xi}_b} = \sum_u \sum_v \frac{\partial \xi_a(t)}{\partial t_u} \frac{\partial \xi_b(t)}{\partial t_v} c\phi v(t_u, t_v) \quad . \tag{3}$$

(We will consistently use the notation $\hat{\sigma}_{\hat{\xi}_a \hat{\xi}_b}$ rather than $c\phi v(\hat{\xi}_a, \hat{\xi}_b)$ to denote the quantity defined by (3).)

The computer obtains numerical values for $\partial\hat{\xi}_a(\underset{\sim}{t})/\partial t_u$ directly by numerical differentiation. It then obtains $cov(t_u, t_v)$ from standard formulas, as will be explained later, and proceeds directly to compute (2) and (3) for all $k(k+1)/2$ pairs in the set $\hat{\xi}_1, \hat{\xi}_2, \ldots, \hat{\xi}_k$.

Asymptotic Hypothesis Testing

In the preceding section, we started with some statistics t_u convenient for defining the $\hat{\xi}_a$. It will not matter what set of statistics we choose so long as the t_u are functionally independent of each other. For (2) and (3), the vector $\underset{\sim}{t} \equiv \{t_u\}$ must include all statistics needed for estimating all parameters in the matrix $\|Cov(t_u, t_v)\|$. For example, we might have $\hat{\xi}_1 = m_1$, $\hat{\xi}_2 = m_2$, and $\|Cov(m_1, m_2)\| = \|\sigma_{ij}\|$, where m denotes a sample mean and σ_{ij} a population covariance. In this case $\underset{\sim}{t}$ must include estimators of the σ_{ij}: $\underset{\sim}{t} = \{m_1, m_2, \hat{\sigma}_{11}, \hat{\sigma}_{12}, \hat{\sigma}_{22}\}$.

In the present section, we start with a set of parameters denoted by $\underset{\sim}{\omega}$ and consider an N-by-n matrix $\underset{\sim}{X}$ of observations drawn from the distribution $f(\underset{\sim}{X}|\underset{\sim}{\omega})$. The parameters $\underset{\sim}{\omega}$ are assumed to be functionally independent of each other. We wish to test the composite hypothesis H_0: $\underset{\sim}{\xi} = \underset{\sim}{0}$, where $\underset{\sim}{\xi} \equiv (\xi_1, \xi_2, \ldots, \xi_k)'$ is a vector of k elements.

Let $\hat{\underset{\sim}{\xi}}$ be an estimate of $\underset{\sim}{\xi}$. If $k = 1$, $\underset{\sim}{\xi}$ is simply a scalar, ξ, and H_0 can usually be tested by computing $\hat{\xi}/\hat{\sigma}_{\hat{\xi}}$ where $\hat{\sigma}^2_{\hat{\xi}}$ is the asymptotic sampling variance of $\hat{\xi}$ with $\underset{\sim}{t}$ substituted for the unknown parameters $\underset{\sim}{\tau}$. The rejection region for H_0 consists of one or both tails of the asymptotic distribution of $\hat{\xi}/\hat{\sigma}_{\hat{\xi}}$ under H_0. In most common problems, this distribution is normal with zero mean and unit variance.

If $\underset{\sim}{\xi}$ is a vector of k elements, H_0 can usually be tested by computing $Q \equiv \hat{\underset{\sim}{\xi}}' \hat{\underset{\sim}{C}}^{-1} \hat{\underset{\sim}{\xi}}$, where $\hat{\underset{\sim}{C}}$ is an estimate of $\underset{\sim}{C} \equiv \|\mathrm{Cov}(\hat{\xi}_a, \hat{\xi}_b)\|$ obtained from (2) and (3). The rejection region for H_0 consists of all large values of Q. In most common problems, the asymptotic distribution of Q is chi square with k degrees of freedom.

Stroud (1971a) proved that the asymptotic hypothesis testing procedure just described is valid under the conditions that

1. The functions $\xi_a \equiv \xi_a(\underset{\sim}{\tau})$ ($a = 1,2,\ldots,k$) have bounded and continuous second derivatives in the neighborhood of $\underset{\sim}{\xi} = \underset{\sim}{0}$.
2. The vector $\underset{\sim}{t}$ is asymptotically normal with mean $\underset{\sim}{\tau}$ and nonsingular $\|\mathrm{Cov}(t_u, t_v)\|$.
3. $\|\mathrm{Cov}(t_u, t_v)\|$ is nonsingular with probability one and converges in probability to $\|\mathrm{Cov}(t_u, t_v)\|$.

These conditions are fulfilled by a broad class of problems, some of which are illustrated in Tables 1 and 2.

If the $\hat{\xi}_a$ are the maximum likelihood estimates of the ξ_a ($a = 1,2,\ldots,k$), obtained without the restriction $\underset{\sim}{\xi} = \underset{\sim}{0}$, then the test described is asymptotically most stringent and is also locally asymptotically most powerful (Moran, 1970; Wald, 1943). A regularity condition worth noting is (as already implied by condition 1 above) that $\underset{\sim}{\xi} = \underset{\sim}{0}$ must not be on the boundary of the range of $\underset{\sim}{\xi}$.

Implementation

As presently written, LASAHT assumes that $f(\underset{\sim}{X}|\underset{\sim}{\omega})$ is multivariate normal. It would be fairly simple to substitute some other assumption, as will be seen. If there are no restrictions on the parameters, the elements of $\underset{\sim}{\omega}$ are presently taken to be the usual parameters: the means (μ_i), variances $(\sigma_{ii}$ or $\sigma_i^2)$, and covariances (σ_{ij}) of the random variables $X_1, X_2, \ldots, X_n$. Naturally, the t_u are taken to be the sufficient statistics: sample means (m_i), sample variances $(s_{ii}$ or $s_i^2)$, and sample covariances (s_{ij}). For asymptotic work, it is immaterial whether the s_{ij} are the unbiased or the usual biased estimators of the σ_{ij}. When there are no restrictions, estimated $\underset{\sim}{\omega}$, denoted by $\hat{\underset{\sim}{\omega}}$, is identical with $\underset{\sim}{t}$; $\underset{\sim}{\omega}$ and $\underset{\sim}{\tau}$ are identical asymptotically.

LASAHT uses standard formulas for the $c\phi v(t_u, t_v)$ required in (2) and (3). The standard formulas for the multivariate normal case, presently incorporated into LASAHT, are

$$\left.\begin{aligned} c\phi v(m_i, m_j) &= \hat{\sigma}_{ij}/N \\ c\phi v(s_{gh}, s_{ij}) &= (\hat{\sigma}_{gi}\hat{\sigma}_{hj} + \hat{\sigma}_{gj}\hat{\sigma}_{hi})/N \\ c\phi v(m_i, s_{gh}) &= 0 \end{aligned}\right\} \qquad (4)$$

where $\hat{\sigma}$ denotes a consistent estimator of a covariance.

When using LASAHT, the statistician specifies k functions $\xi_1(), \xi_2(), \ldots, \xi_k()$ in which he is interested. He does this simply by

writing a FORTRAN arithmetic assignment statement for each function, expressing the corresponding $\hat{\xi}_a$ as a function of $\underset{\sim}{t}$. He inserts these statements in a place provided in the program. LASAHT proceeds automatically from this point, computing $\underset{\sim}{t}$, $\hat{\underset{\sim}{\xi}} \equiv \{\hat{\xi}_a\} \equiv \{\xi_a(\underset{\sim}{t})\}$, $\|\partial\hat{\xi}_a/\partial t_u\|$, $\|c\hat{o}v(t_u,t_v)\|$, $\hat{\underset{\sim}{C}} \equiv \|\hat{\sigma}_{\hat{\xi}_a\hat{\xi}_b}\|$, $Q \equiv \hat{\underset{\sim}{\xi}}'\hat{\underset{\sim}{C}}^{-1}\hat{\underset{\sim}{\xi}}$, and finally the percentile rank of Q in the appropriate chi square distribution.

If $f(\underset{\sim}{X}|\underset{\sim}{\omega})$ is not multivariate normal, it is only necessary to redefine the t_u as functions of the observations $\underset{\sim}{X}$ and to replace (4) by correct formulas for $\|c\hat{o}v(t_u,t_v)\|$. With these changes, LASAHT can proceed just as described.

Without user action, the program accommodates two samples, each composed of any number of observations on a maximum of 10 random variables. More samples (up to 20) with fewer random variables can be accommodated if the user sets all population covariances between variables from different samples equal to zero (see below). In addition, the maximum of 10 random variables per sample can be increased, if desired.

Restrictions on the Parameters

In the normal multivariate case, there is a total of $n(n+3)/2$ sample means, variances, and covariances. Unless instructed otherwise, LASAHT automatically uses these $n(n+3)/2$ sample statistics as estimators for the corresponding parameters in $\underset{\sim}{\omega}$.

If r restrictions are imposed on the parameters (for example, certain means or variances are known to be equal, or certain covariances are known to be zero), then K, the number of parameters in $\underset{\sim}{\omega}$, is

correspondingly reduced. If r restrictions are imposed in the normal multivariate case, then $K = n(n + 3)/2 - r$. When r restrictions are to be imposed, the statistician must arrange matters so that r of the estimators are functions of the remaining K independent estimators, to be denoted by $\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_K$.

He does this by inserting in the program FORTRAN arithmetic assignment statements defining whatever estimators he wishes. For example, if it is known that two population means μ_1 and μ_2 are equal, the statistician would supply a FORTRAN statement defining $\hat{\underset{\sim}{\xi}}$ in terms of $\hat{\mu}_1$, $\hat{\mu}_2$, and other estimators. If, for example, the sample sizes are equal, he could then insert in the program the FORTRAN equivalent of the definitions $\hat{\mu}_1 = (m_1 + m_2)/2$ and $\hat{\mu}_2 = (m_1 + m_2)/2$.

In this way the $\hat{\xi}_a$ and the $\hat{\omega}_u$ are directly or indirectly defined as functions of the m_i and the s_{ij}. It will be convenient to refer to the m_i and the s_{ij} collectively as the T_p, $p = 1,2,\ldots, n(n + 3)/2$.

By (3),

$$\mathrm{cov}(\hat{\omega}_u, \hat{\omega}_v) = \sum_p \sum_q \frac{\partial \hat{\omega}_u}{\partial T_p} \frac{\partial \hat{\omega}_v}{\partial T_q} \mathrm{cov}(T_p, T_q) \quad , \tag{5}$$

where $\mathrm{cov}(T_p, T_q) \equiv \mathrm{Cov}(T_p, T_q)\Big|_{\omega = \hat{\omega}}$. Replacing the t_u in (3) by $\hat{\omega}_u$ and using (5), we have by the chain rule for differentiation

$$\hat{\sigma}_{\hat{\xi}_a \hat{\xi}_b} = \sum_u \sum_v \frac{\partial \xi_a}{\partial \hat{\omega}_u} \frac{\partial \xi_b}{\partial \hat{\omega}_v} \sum_p \sum_q \frac{\partial \hat{\omega}_u}{\partial T_p} \frac{\partial \hat{\omega}_v}{\partial T_q} c\phi v(T_p, T_q)$$

$$= \sum_p \sum_q \frac{\partial \xi_a}{\partial T_p} \frac{\partial \xi_b}{\partial T_q} c\phi v(T_p, T_q) \quad . \tag{6}$$

The $c\phi v(T_p, T_q)$ are given by (4). LASAHT conveniently obtains the $\hat{\sigma}_{\hat{\xi}_a \hat{\xi}_b}$ from (6) rather than from (3).

Example 1

Suppose the statistician wishes to test the hypothesis $\xi \equiv \sigma_{12} = 0$ under the restriction $\mu_1 = \mu_2$. For the normal bivariate case,

$$\underset{\sim}{T} \equiv \{T_p\} = \{m_1, m_2, s_1^2, s_{12}, s_2^2\} \quad .$$

It does not matter whether $\underset{\sim}{\omega}$ is defined as $\{\mu_1, \sigma_1^2, \sigma_{12}, \sigma_2^2\}$ or as $\{\mu_2, \sigma_1^2, \sigma_{12}, \sigma_2^2\}$. The statistician supplies the FORTRAN definitions

$$\left.\begin{aligned} \hat{\mu}_i &\equiv (m_1 + m_2)/2 \quad , \\ \hat{\sigma}_i^2 &\equiv s_i^2 + m_i^2 - \hat{\mu}_i^2 \end{aligned}\right\} \qquad (i = 1,2) \quad ,$$

and

$$\hat{\sigma}_{12} \equiv s_{12} + m_1 m_2 - \hat{\mu}_1 \hat{\mu}_2 \quad ,$$

$$\hat{\xi} = \hat{\sigma}_{12} \quad .$$

LASAHT proceeds automatically from this point on. In accordance with (6), Var $\hat{\xi}$ is computed from the $\mathrm{cov}(T_u, T_v)$ given by (4).

The $\hat{\mu}$ and $\hat{\sigma}$ given above are well known as the maximum likelihood estimators under the restriction $\mu_1 = \mu_2$. LASAHT does not automatically obtain MLE. If the statistician does not have formulas for the MLE, he can use any consistent estimators in their place. In example 1, he could without much loss have chosen $\hat{\sigma}_1^2 \equiv s_1^2$, $\hat{\sigma}_2^2 = s_2^2$, $\hat{\sigma}_{12} = s_{12}$. If inefficient estimators are used, the test of the hypothesis is still valid, but the power of the test is reduced.

Illustrative Problems

LASAHT has been checked out by applying it to numerical examples, testing some three dozen different null hypotheses for which the numerical answers could be verified. The partial listing in Tables 1 and 2 may suggest the scope of the program. Primes are used to distinguish parameters of two different populations; $\rho_{ij} \equiv \sigma_{ij}/\sigma_i\sigma_j$, $\underset{\sim}{\mu} \equiv (\underset{\sim}{\mu}_1, \underset{\sim}{\mu}_2) \equiv (\mu_i)$, and

$$\underset{\sim}{\Sigma} \equiv \begin{bmatrix} \underset{\sim}{\Sigma}_{11} & \underset{\sim}{\Sigma}_{12} \\ \underset{\sim}{\Sigma}_{21} & \underset{\sim}{\Sigma}_{22} \end{bmatrix} \equiv \|\sigma_{ij}\| \quad .$$

Table 1

Illustrative Problems, k = 1

Null Hypothesis	Restrictions	
$\sigma_1 = \sigma_2$	$\sigma_{12} = 0$	
$\rho_{12} = 0$	$\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$	
$\frac{1}{2}[\log(1 + \rho) - \log(1 - \rho)] = \text{constant}$	none	
$\rho_{12}\rho_{34} - \rho_{13}\rho_{24} = 0$	none	
$\sigma_{12}\sigma_{34} - \sigma_{13}\sigma_{24} = 0$	$\sigma_{13} = \sigma_{14}$, $\sigma_{23} = \sigma_{24}$, $\sigma_3 = \sigma_4$	
$\frac{\rho_{12}\rho_{34} - \rho_{13}\rho_{24}}{\sqrt{(\rho_{34} - \rho_{13}^2)}\sqrt{(\rho_{34} - \rho_{24}^2)}} = 0$	same	(Lord, in press)
$\mu_1' = 0$ (both populations are bivariate)	$\mu_2' = \mu_2''$, $\sigma_2' = \sigma_2''$	(Lord, 1955)
trace $\Sigma^{-1} = \text{constant}$	none	
$\lvert\Sigma\rvert = \text{constant}$	none	(Madansky and Olkin, 1969)
trace$(\Sigma'^{-1}\Sigma'') = \text{constant}$	none	

Table 2

Illustrative Problems, $k > 1$

Null Hypothesis	Restrictions	
$\mu = 0\ ,\quad \sigma^2 = 1$	none	
$\mu_1 = \mu_2 = 0$	none	
$\sigma_1^2 = \sigma_2^2 = 1$	none	
$\sigma_1^2 = 1\ ,\quad \sigma_{12} = .5$	none	
$\underset{\sim}{\Sigma}'_{11} - \underset{\sim}{\Sigma}'_{12}(\underset{\sim}{\Sigma}'_{22} - \underset{\sim}{I})^{-1}\underset{\sim}{\Sigma}'_{21} = \underset{\sim}{\Sigma}''_{11} - \underset{\sim}{\Sigma}''_{12}(\underset{\sim}{\Sigma}''_{22} - \underset{\sim}{I})^{-1}\underset{\sim}{\Sigma}''_{21}$	none	(Stroud, 1971b)
$\left.\begin{array}{l} \underset{\sim}{\Sigma}'_{12}(\underset{\sim}{\Sigma}'_{22} - \underset{\sim}{I})^{-1} = \underset{\sim}{\Sigma}''_{12}(\underset{\sim}{\Sigma}''_{22} - \underset{\sim}{I})^{-1} \\ \underset{\sim}{\mu}'_1 - \underset{\sim}{\Sigma}'_{12}(\underset{\sim}{\Sigma}'_{22} - \underset{\sim}{I})^{-1}\underset{\sim}{\mu}'_2 = \underset{\sim}{\mu}''_1 - \underset{\sim}{\Sigma}''_{12}(\underset{\sim}{\Sigma}''_{22} - \underset{\sim}{I})^{-1}\underset{\sim}{\mu}''_2 \end{array}\right\}$	none	(Stroud, 1971b)

Example 2

Consider testing the null hypothesis that the tetrad $\rho_{12}\rho_{34} - \rho_{13}\rho_{24} = 0$, with the restrictions that $\sigma_3 = \sigma_4$, $\sigma_{13} = \sigma_{14}$, $\sigma_{23} = \sigma_{24}$ (Lord, in press). First, let us replace the null hypothesis by the equivalent hypothesis that $\xi \equiv \sigma_{12}\sigma_{34} - \sigma_{13}\sigma_{24} = 0$. This definition of the function ξ is provided to the computer by inserting in the program the arithmetic assignment statement

XIHAT(1) = [FORTRAN equivalent of $\hat{\sigma}_{12}\hat{\sigma}_{34} - \hat{\sigma}_{13}\hat{\sigma}_{24}$] .

It is further necessary to provide FORTRAN statements defining the $\hat{\sigma}_{ij}$ in terms of sample means, variances, and covariances. We choose

$$\hat{\sigma}_{13} = \hat{\sigma}_{14} = (s_{13} + s_{14})/2 \quad ,$$

$$\hat{\sigma}_{23} = \hat{\sigma}_{24} = (s_{23} + s_{24})/2 \quad ,$$

$$\hat{\sigma}_3^2 = \hat{\sigma}_4^2 = (s_3^2 + s_4^2)/2 \quad ,$$

which are the maximum likelihood estimators under the stated restrictions. The estimators $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$, and $\hat{\sigma}_{12}$ are not defined explicitly in the program, with the result that the computer resorts to a default procedure that assumes (correctly) that $\hat{\sigma}_1^2 = s_{11}$, $\hat{\sigma}_2^2 = s_{22}$, $\hat{\sigma}_{12} = s_{12}$, and $\hat{\sigma}_{34} = s_{34}$.

Provided with the FORTRAN definitions shown, the computer will now compute $\hat{\xi}$, its estimated asymptotic variance $\hat{\sigma}_{\hat{\xi}}$, the test statistic $\hat{\xi}/\hat{\sigma}_{\hat{\xi}}$, and the percentile corresponding to the test statistic. All this is easier for the statistician than deriving the formula for $\hat{\sigma}_{\hat{\xi}}^2$ --an

eighth degree polynomial containing ten terms involving seven statistics--and then computing the test statistic from this formula.

Example 3

In a Monte Carlo study, 1000 values of $\hat{\xi}/\hat{\sigma}_{\hat{\xi}}$ and their probability levels were computed by LASAHT, where

$$\hat{\xi} = \frac{\hat{\rho}_{12}\hat{\rho}_{34} - \hat{\rho}_{13}\hat{\rho}_{24}}{\sqrt{(\hat{\rho}_{34} - \hat{\rho}_{13}^2)}\sqrt{(\hat{\rho}_{34} - \hat{\rho}_{24}^2)}}$$

(Lord, in press). The time required on a 360/65 for all 1000 was about 80 seconds.

Example 4

The last example in Table 2 was carried through for sets of data (Stroud, 1971b) having 8 observed variables in each of two groups. Thus there were 88 different sample statistics T_p involved in the double summation in (6). The vector null hypothesis $\underset{\sim}{\xi}$ that was tested consisted of $k = 18$ separate equations of the form $\xi_a = 0$. In three separate applications, LASAHT produced values for the test statistic identical to those obtained by Stroud using complicated analytic formulas.

The hypothesis tested may be described as a multivariate analysis of covariance hypothesis with three criterion variables and five covariables, modified to take account of random errors of measurement in the covariables. Problems of this complexity are very difficult to carry through without the aid of a program such as LASAHT.

Acknowledgments

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